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A
METHOD
OF
COMPUTING ASTRONOMICAL REFRACTIONS
FOR
OBJECTS NEAR THE HORIZON.

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ALTHOUGH observations of zenith distances, when the object is near the horizon, are frequently affected by irregularities of refraction, that seem not capable of being reduced to any law, yet there is reason to suppose that the effect of these irregularities will disappear in a mean of a great number of observations; and thus a mean refraction for any altitude may be obtained, depending only on the mean zenith distance, and the corresponding heights of the barometer and thermometer. The investigation of the law of these *regular* refractions, as they may be called, has much engaged the attention of astronomers.

This inquiry has led to the extremely complex but elegant mathematical researches of Kramp, Laplace, and Bessel. Their investigations are nearly related to each other. Dr. Young* has also recently, by an entirely different method, and with great analytical skill, obtained an equation expressing the relation between the refractive force of air and the refraction at any zenith distance.

It is an objection to these inquiries, however curious in themselves, that the results are entirely useless for the nicer purposes of astronomy, unless we shall also be able to reduce to calculation what now appear as irregularities.

To this it may be answered, that, when the difficulties as to the regular refractions are overcome, the way will be cleared for an attempt, apparently indeed more hopeless, on the others.

It is admitted, that the true refraction is always less than that computed on the hypothesis of an uniform temperature, and greater than that obtained by supposing the density to decrease uniformly; that, as far as 80° from the zenith, these limits approach each other very closely†; and that, till very near the horizon, they do not widely recede from each other.

The former hypothesis has given occasion to elaborate investigations by M. M. Kramp and Laplace, and a modification of it to still more elaborate ones by M. Bessel.

It is the object of this paper to deduce, by help of a modification of the result of the hypothesis of a density decreasing uniformly, by an extremely simple investigation, the refraction, at any low altitude, *corresponding to any heights of the barometer and thermometer.* The Tables thence resulting for zenith distances, between 80° and the horizon, will, I conceive, be found as convenient as can be desired. They scarcely yield in simplicity to the French Tables, and enable us to obtain the quantity of refraction, as changed by the weight and temperature of the atmosphere, in which, near the horizon, the French Tables appear entirely to fail.

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* Phil. Trans. 1819, Part I.

† Trans. R. I. Academy, vol. 12. p. 89.

The first Tables in which this has been attended to, as far as the horizon, if I mistake not, were those of Mr. Bessel.[*]

In our ignorance of the law of variation of density, we can only verify any hypothesis that we adopt, by a comparison of its results with those obtained by direct observation. In this way, by help of Dr. Bradley's observations, Mr. Bessel has obtained a modification of the law of uniform temperature, that will give the refractions, to within about three degrees of the horizon, with great exactness. Dr. Young has, by adopting a law of variation of temperature advanced by Professor Leslie, obtained an equation for refraction, the solution of which gives the refractions with considerable exactness as far as the horizon.

The following method is derived from the formula obtained in the hypothesis of a density decreasing uniformly.

It is shown in my paper,[†] above refered to, that, on this hypothesis,

$$\text{Refr.} = \frac{m-1}{\sin 1''} \tan \left[\theta - \left(\frac{l}{a\,(m-1)} - \frac{1}{2} \right) \text{refr.} \right]$$

Where θ = zenith distance, l = height of uniform atmosphere, a = radius of earth, and $m : 1 ::$ sin incid. : sin refr. for air at the surface; or making $S = \left(\frac{l}{a\,(m-1)} - \frac{1}{2} \right)$ refr. we may suppose the true refraction $= \frac{m-1}{\sin 1''} \tan (\theta - k\,S)$, the multiplier k as well as S being different for each zenith distance.

Or making $\theta - k\,S = \theta'$, $S = \left(\frac{l}{a(m-1)} - \frac{1}{2} \right) \frac{m-1}{\sin 1''} \tan \theta' = \left(\frac{l}{a} - \frac{m-1}{2} \right) \frac{\tan \theta'}{\sin 1''}$

$$\text{and the true refraction} = \frac{(m-1)}{\sin 1''} \tan \left[\theta - \frac{k \tan \theta'}{\sin 1''} \left(\frac{l}{a} - \frac{m-1}{2} \right) \right]$$

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[*] Bessel's Fundamenta Astron. p. 26, &c.

[†] Trans. R. I. Academy, Vol. 12, p. 94.

From the zenith distance to about 80°, $k = 1$ and at 90° $= 0, 8$ nearly, but it seems reasonable to suppose, that, whatever be the law of variation of density, which occasions this quantity k to differ from unity, that k is constant, or very nearly so, at the same zenith distance, whatever may be the change of density at the surface. For, however great within the usual limits we may suppose the change of density at the surface to be, there is no reason to suppose a material change in the *law* of density in the atmosphere.*

Hence, if we obtain the quantity $\frac{k \tan \theta'}{\sin 1}\left(\frac{l}{a} - \frac{m-1}{2}\right)$ for any zenith distance and density at the surface, we shall easily obtain it for the same zenith distance and any other density, by considering k and $\tan \theta'$ as constant. It is evidently sufficient to take the mean value of θ'

$$m - 1 = (m' - 1) \times \frac{\text{barom.}}{29.60} \times \left(1-(t-50),0001\right) \times \frac{1,0375}{1+,002083\ (t-32)} \quad \text{and}$$

$$\frac{l}{a} = \frac{l'}{a} \times \frac{1+,002083\ (t-32)}{1,0375} \dagger$$

where $m' - 1$ and $\frac{l'}{a}$ are the mean values of $m - 1$ and $\frac{l}{a}$ corresponding to Far. Therm. 50 and Barom. $\overset{in.}{29,60}$ and which, adapted to the French Tables, are

$$m' - 1 = 57'',72 \sin 1'' \text{ and } \frac{l'}{a} = ,00128$$

Hence the mean value of

$$\frac{k\, l \tan \theta'}{a \sin 1''} = 264'',02\ k \tan \theta' \text{ and of } \frac{k\,(m-1) \tan \theta'}{2 \sin 1''} = 28'',86\ k \tan \theta'$$

By help of a great number of observations of a given star, we can obtain the refraction corresponding to the mean of the Barometers and mean of the Thermometers, and thus obtain the value of *k S* corresponding to the zenith distance of that Star, and hence its mean value for Barom. $\overset{in.}{29{,}60}$ and Therm. 50.

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* This reasoning may be fallacious, and it appears to be very desirable, that the facts should be ascertained by a sufficient number of observations, at given zenith distances near the horizon, for different values of $m - 1$.

† Trans. R. I. Acad. Vol. 12, p. 98, 99.

By selecting 3 or 4 stars between 88° and 90° zenith distance, and 3 or 4 between 80° and 88° zenith distance, the mean values of *k S* may be determined, (if a sufficient number of observations be used to make the effects of the irregular refractions disappear.) Then a Table of the mean values of *k S* between 80° and 90° zenith distance, may be interpolated. From the mean value for any zenith distance obtained by this Table, the actual value of *k S* may be computed from the heights of the Barometer and Thermometer, and then the refraction is easily had.

By this method a Table may be much more readily constructed, and, if I mistake not, with greater exactness, than by help of any hypothesis of the actual variation of density.

Or this method may be applied to the verification of any Table of refractions. From such a Table, we can readily find, by computation, $\frac{k \tan \theta'}{\sin. 1'} \left(\frac{l'}{a} - \frac{m'-1}{2} \right)$ for each zenith distance.

The parts of this quantity have a given ratio to each other, whatever be the zenith distance; therefore each can be readily deduced from the whole, and the first may be put into a convenient Table. From this Table we may readily find for any height of the Barometer and Thermometer,

$$k \tan \theta' \left(\frac{l}{a \sin. 1''} \right) \text{ and } k \tan \theta' \left(\frac{m-1}{2 \sin. 1'} \right)$$

by substituting for l and m their values above given.

Their difference $= k\,S$ and the refraction $= \frac{m-1}{\sin. 1''} \tan (\theta - k\,S)$ being compared with the observed, will easily show the correction that *k S* requires; and thence the correction required by the mean values. For this examination, as was said, a mean of a very considerable number of observations will be required, for each of the zenith distances that shall be considered necessary to be examined for the verification of the Table,

The Table I have adopted for computing the mean values of log. $\frac{k\,l \tan \theta'}{a \sin. 1''}$, (vid. my Table I.) is the French Table of mean refractions. It appears to me, that for zenith distance very near the horizon, more weight is to be given to that than to any other we possess. The well known accuracy, skill, and judgment of M. Delambre, seem justly entitled to this confidence. He tells us he constructed it by help of many hundred observations made by a repeating circle from 70° to 90° 20′.

But it must be confessed, that reference can no where be had to a sufficient number of observations to verify the numbers in my Table I. corresponding to any one zenith distance between 88° and 90°. Mr. Groombridge, to whom we are much indebted for his numerous observations of Stars at low altitudes, was unable, from local circumstances, to observe much below 88°; above 88°, several references may be had to observations sufficiently numerous, (among others those of Dr. Bradley, computed by Mr. Bessel,) by which it will appear that Table I. is very exact, and consequently the French Table of mean refractions from which it was deduced.

When however it is required to compute the refraction for other states of the Barometer and Thermometer, the results by my tables and the French differ for low altitudes. This is occasioned from it being supposed in the French Tables that the refraction is proportional to the density, which is by no means sufficiently exact in very low altitudes.

The investigations of M. Laplace, on which the French tables are founded, are indeed exact on this head; but the difficulty of introducing the variation into the tables seems to have occasioned its omission. M. Bessel's investigations are rendered more intricate, by developing this change in a manner so as to be conveniently introduced into tables. In my method the development and tables appear equally simple:

	Inches			′ ″
At 87° Z. D.	Bar. 28,00	the mean ref. is	by French Tables	1.19,2
	Therm. 70°	decreased	by my Tables	1.29,1
At 88° Z. D.	Bar. 30,50	the mean ref. is	by French Tables	1.26,6
	Therm. 25	increased	by my Tables	2. 4,0
At 90° Z. D.	Bar. 29,60	refraction	by French Tables	34′.42″
	Therm. 32°		by mine	36.18
Whereas the mean refraction is by each				33.23

At present we have not sufficient observations to determine, whether the actual variations of refractions at low altitudes are most conformable to the theory of M. Bessel, to that of Dr. Young, or to that above given. It can only be ascertained by help of very numerous observations, so numerous that they must require the cooperation of many observers; but the exactest instruments are not required, and much assistance might be given by instruments of inferior accuracy, and of which the errors may amount to some seconds.

In the following Tables:

$$\text{Tab. I.} = \log k \left(\frac{l'}{a \sin 1''}\right) \tan \theta'$$

$$\text{Tab. II.} = \log \left[\frac{1{,}0375}{1 + {,}002083\,(t - 32)} \times \left(1 - {,}0001\,(t - 50)\right) \times \frac{\overset{\prime\prime}{57{,}72}}{29{,}60}\right]$$

$$\text{Tab. III.} = \log \left(\left(1 - {,}0001\,(t - 50\right) \times \frac{\overset{\prime\prime}{57{,}72}}{29{,}60}\right)$$

USE OF THE TABLES.

Log A in minutes of a degree = Tab. I. + ar. comp. Tab. II. + Tab. III.

Log B in minutes = Tab. I. + $\log \frac{28,86}{204,02}$ + Tab. II. — log. 57,72 + log. barom. = Tab. I. + Tab. II. + log. barom. + 7.2773.

Log refraction in seconds = Tab. I. + log barom. + log tan (zen. dist. — A + B)

If Table II. p. 103. vol. 12. Trans. R. I. Academy be added to these they will serve for computing the refraction from the zenith to the horizon.

Example.

App. zenith dist. (of α Lyræ S. P.) = 87° 42′.10″ Bar. = 29,50 & Therm. 35°,0*

Tab. I. for 87°.42,′17 gives ... 1 8160
Tab. II. for Therm. 35° ... 0.3039
Tab. III. 0.2906

Tab I. 1.8160
a. c. Tab. II. 9.6961
Tab. III. 0.2906
A — 63′,49 log 1.8027
B + 7′,36
— 56,13
87 42,17
(Z D — A + B) 86.46,04 tan 11.2481
Tab. II. 0.3039
log Bar. 1.4698
refr. 1051″,5 log 3.0218
= 17′31″,5

Tab. I. 1.8160
Tab. II. 0.3039
log Bar. 1.4698
const. 7.2773
B 7′,36 log 0.8670

The mean of 42 observations of α Lyræ S. P. made at the observatory of Trinity College Dublin, (mean of bar. 29,50 and mean of therm. 35°) gave 17′.26′,5.

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* Trans. R. I. Acad. vol. 12. p. 112

TABLE I.

Zenith Dist	Logarithms.	Diff. for 1′.
° ′		′
80.0	1 3605	6,90
81.0	1.4031	7,62
82 0	1.4488	8,21
83.0	1.4981	9,07
84.0	1.5524	10,13
85.0	1.6132	11,00
85.30	1 6462	11,37
86. 0	1.6803	12,07
86.30	1.7165	13,23
87. 0	1.7562	13,75
87.20	1.7837	14,55
87.40	1.8128	14,65
88 0	1.8421	15,50
88.20	1.8731	16,05
88.40	1.9052	16,75
89. 0	1.9387	17,25
89.20	1.9732	18,00
89.40	2.0092	18,60
90. 0	2.0464	

TABLE II.

Far. Therm	Logarithms	Far. Therm.	Logarithms	Far. Therm.	Logarithms.
10	0.3283	34	0.3048	58	0.2827
11	0.3273	35	0.3039	59	0.2818
12	0.3263	36	0.3030	60	0.2809
13	0.3253	37	0.3020	61	0.2800
14	0.3243	38	0.3011	62	0.2791
15	0.3233	39	0.3001	63	0.2782
16	0 3223	40	0.2992	64	0.2773
17	0.3213	41	0.2983	65	0.2764
18	0 3203	42	0.2974	66	0.2755
19	0.3193	43	0.2965	67	0.2746
20	0.3183	44	0.2956	68	0.2737
21	0.3173	45	0.2946	69	0.2728
22	0.3163	46	0.2937	70	0.2720
23	0.3154	47	0.2928	71	0.2711
24	0.3144	48	0.2919	72	0.2703
25	0.3134	49	0.2910	73	0.2694
26	0.3124	50	0.2900	74	0.2685
27	0.3114	51	0.2891	75	0.2677
28	0.3105	52	0.2881	76	0.2668
29	0.3095	53	0.2872	77	0.2660
30	0.3086	54	0.2863	78	0.2652
31	0.3076	55	0.2854	79	0.2644
32	0.3067	56	0.2845	80	0.2636
33	0.3058	57	0.2836	81	0.2627

TABLE III.

Therm. near Baróm.	Logarithms.
°	
20	0.2913
30	0.2909
40	0.2904
50	0.2900
60	0.2896
70	0.2891
80	0.2887